Solving the generalized non-linear Schrödinger equations with genetic algorithms

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Genetic algorithms(GAs) are optimization algorithms that take inspiration from Darwin's natural selection principle. GAs have applications in areas like image processing, information retrieval, grammar induction, data mining, natural language processing, solving nonlinear equations and others[1]. This algorithm emulates the Darwinian principle to solve mathematical and optimization problems. The objective of the algorithm is to find a good solution to the problem by generating an environment where a population of possible solutions to a problem can be generated, combined and evaluated (using a fitness function as a benchmark). By repeating the Darwinian selection over time, the algorithm tries to generate better solutions. There are three main steps that a genetic algorithm follows in order to solve a problem: generation of an initial random population, creation of a successive population by reproduction, crossing, and mutation, finally, after several iterations of step 2, the best candidate solution is found from the last population[2–4]. Figure 1 shows the steps followed by a simple GA.

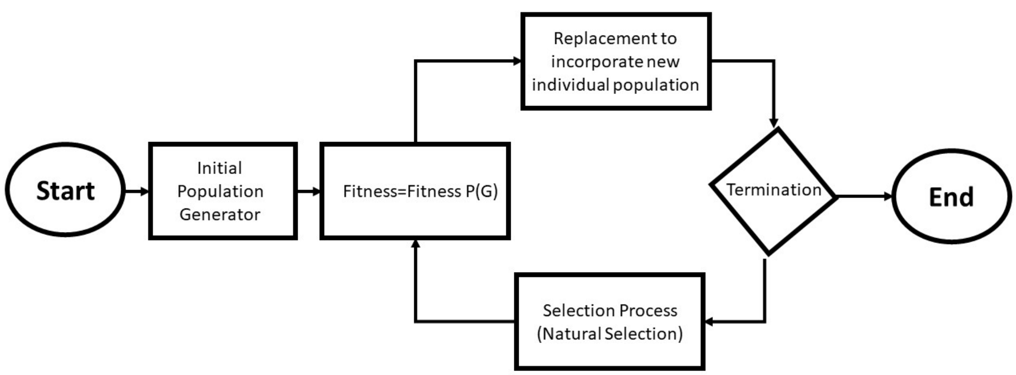

Figure 1: Diagram of a simple Genetic algorithm

One of the applications of GAs is solving the generalized nonlinear Schrödinger equation (GNLSE) in optics. Solitons are self-trapped beams in nonlinear media and have potential applications in optic communication systems and photonic technologies. Theoretical models for generating solitons usually do not admit exact solutions as they rely on the GNLSEs. A general approach for solving this type of equations does not exist. Numerical methods like the standard relaxation algorithm and conjugate gradient method are effective but they are not always enough[5].

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The GNLSE equation that the GA must solve has the form:

$$-\lambda\Psi + \nabla^2\Psi + |\Psi|^2\Psi = 0,$$

Where Ψ is the wave equation of the soliton/optical field, λ is a propagation constant and ∇^2 is the transversal lagrangian ($\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$).

The aim of this work is creating an algorithm capable of finding the parameters (amplitude and width of the soliton) that approximate the left side of the equation to zero. After the GA approximates the solution a relaxation algorithm could be used to find a more exact solution. This is needed so that Ψ approximates best the behavior of the soliton. The GAs algorithm approximates the solution by taking parameter in the equation as genes, then it mutates and reproduces the parameters so that the error is minimized. The fitness function is:

$$-\lambda\Psi + \nabla^2\Psi + |\Psi|^2\Psi = E,$$
$$Error = \iint_{-\infty}^{\infty} EE^* \, dx \, dy,$$

The fitness function tries to minimize the error. The technical contributions of the presenting author include the design and implementation of the GA. Until now, algorithms need conditions close to the solution as input[6]. The advantage of the presented algorithm is the lack of concern from providing these conditions because the GA automatically optimizes them. The final objective of this work is providing an algorithm that can solve GNLSE equations for more than 100 optical profiles of solutions (amplitudes and widths).

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