Mapping the loss of information of Bosonic (Physical) systems into neural networks with applications in Machine learning

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Abstract

We study the loss of information of a Bosonic physical system from the perspective of neural networks. We then analyze the evolution of the Hamiltonian, finding in this way the conditions under which the synapse connecting the neurons changes its behavior such that no more information is allowed to cross through the system. This study have important applications in Machine learning in order to increase the efficiency of the transmission of information and in order to avoid possible loss of information and loss in the ability of learning.

1 Introduction

If we intend to analyze the loss of information in a neural network arrangement, inspired in physical systems, a natural approach consists in mapping the degrees of freedom of a Bosonic system into the degrees of freedom of a neural network arrangement. Several attempts of using this technique have been proposed recently [1]. A standard technique able to analyze the loss of information in neural networks by using this approach was then required. The Bosonic fields store the information inside Quantum Fields and then by unitary evolution, the information is normally preserved [2]. From the perspective of neural networks, by taking the information as a Quantum Field, this is equivalent to say that the map of information from a group of neurons toward others, is done by using Bogoliubov transformations. The loss of information happens when the Bogoliubiov map is non-unitary. A typical example of this was analyzed in [3] by taking a Black-Hole as an example of a Bosonic system. In such a case the Black-Hole evaporation or equivalently, the Hawking radiation, is a typical example of loss of information in a Bosonic system [4]. In this paper we explain the general technique for the analysis of loss of information in neural networks by using the Bogoliubov method. In addition, we explain how the loss of information can in some circumstances produce total loss in the ability of transmission of the information through the synapses connecting the different neurons. Finally we explain the relevance of this analysis in Machine Learning. In fact, understanding how the information is loss inside a system, can help us to develop techniques for avoiding such scenarios. Note that the loss of information is an effect that in most of the cases cannot be avoided but rather minimized. In addition, the loss of information is different to the minimization of a cost function. However, both quantities can be connected Mathematically.

2 Bosonic field mapped to a neural network arrangement

Consider a Bosonic field obeying the usual commutation relations defined as

$$[\hat{a}_i, \hat{a}_j^+] = \delta_{ij}, \qquad [\hat{a}_i, \hat{a}_j] = [\hat{a}_i^+, \hat{a}_j^+] = 0 \tag{1}$$

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The previous commutators define a Bosonic algebra. Whenever we use it, we can imagine that we are dealing (up to some level) with the Quantum Harmonic oscillator. It is in this way how the Quantum scalar fields are analyzed [2]. For mapping the degrees of freedom of from the Quantum field to the neural network arrangement, we use the following convention: 1). The gap between the occupation state $n_k >$ and $|n_k \pm 1$ is equivalent to the effort necessary for storing the information inside a neuron through the gap expression $E_{n_0,n_1,...n_K} = \sum_n E_k n_k$, with $K+1$ marking the number of neurons in the system. 2). The index k in the occupation number n_k corresponds to the label for each neuron under analysis. 3). The information is considered to be stored through the patterns of the occupation number $|n_0 n_1 ... n_K >$, which is the tensorial product of the neuron states corresponding to the same layer if we are working inside the scenario of Deep Learning. 4). The connection between neurons or synapses in biological terms, appear as a coupling between neuron states in a Hamiltonian. This will correspond to the weights for the coupling between neurons. Having this information about neural networks, we can safely formulate its Hamiltonian in the following standard form [1, 3]

$$\hat{H} = \sum_{k=1}^{K} E_k \left(1 - \alpha \hat{n}_0\right) \hat{n}_k + E_0 \hat{n}_0. \tag{2}$$

The Hamiltonian is related to the gap or energy invested in order to store or transmit information through the network. The eq. (2), illustrates the connection or coupling between a single neuron $\hat{n}_0$ and several neurons $\hat{n}_k$. In the standard language of neural networks, the weights used for defining the importance of some instruction [5], would be defined as $\omega_k = -E_k \alpha$ in agreement with eq. (2). The negative sign means that if $\alpha > 0$, then there is a favorable flow of information between the connected neurons. If the sign of this term is inverted, the flow of information will be more difficult to make since this would correspond to an unfavorable effect. Note that the Hamiltonian of the free-neurons (without any synapse or connection) would be $\hat{H}_{free} = \sum_k E_k \hat{n}_k$, which is the standard energy level of the Harmonic oscillator omitting the energy of the ground state. If we have multiple connections between neurons, then we would have additional terms of the form $\omega_k = E_k \alpha_z$ (with an additional label for α). This case would correspond to a trivial extension of the Hamiltonian (2). Now we will proceed to explain how we can analyze the loss of information in neural networks and what are the general consequences for Machine learning.

3 The loss of information in the arrangement: Bogoliubov technique

The information from a neuron, propagating toward the next neuron through a synapse, can be modelled by a Bogoliubov transformation. If the information is preserved, the transformation will preserve the commutators and it will be unitary. If the information is not preserved, then we have a non-unitary transformation, similar to the Mathematical arrangement for deriving the Black-Hole evaporation or Hawking radiation [4]. In this way, if we define the excitation number as $\hat{n}_i = \hat{a}_i^+ \hat{a}_i$, which is related to the expansion of a Quantum scalar field in the form

$$\phi_1(x) = \sum_i \left(e^{-ikx} \hat{a}_i + e^{ikx} \hat{a}_i^+\right), \tag{3}$$

Either, the Quantum scalar field or the excitation number, help us to define the amount of information stored inside a neuron. When the information is transmitted toward another neuron, then we can make a map of the modes represented in eq. (3) into a new set of modes given by

$$\phi_2(x) = \sum_i \left(e^{-ikx} \hat{b}_i + e^{ikx} \hat{b}_i^+\right), \tag{4}$$

with the corresponding occupation number $\hat{n}_i = \hat{b}_i^+ \hat{b}_i$. In the most general situations, the modes are related by the transformation $\hat{b}_i = \sum_j \left(\bar{\alpha}_{ij} \hat{a}_j - \bar{\beta}_{ij} \hat{a}_j^+\right)$. If the information is preserved during the transmission, then $\bar{\beta}_{ij} = 0$ and the modes in eq. (3) are equivalent to those defined in eq. (4). However, if $\bar{\beta}_{ij} \neq 0$, then part of the transmitted information is lost. This project intends to explain how this generic loss, if not controlled, can create the scenario where it becomes impossible to transmit information. The same scenario also creates the impossibility of learning. It can be demonstrated that this process of losing information generates in the long term in-capabilities for learning. Part of this research is already published and part is in process.

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