## **Lossless Compression of Deep Neural Networks**

Thiago Serra<sup>1</sup> Abhinav Kumar<sup>2</sup> Srikumar Ramalingam<sup>3</sup>

## Abstract

Deep neural networks have been successful in many predictive modeling tasks, such as image and language recognition, where large neural networks are often used to obtain good accuracy. Consequently, it is challenging to deploy these networks under limited computational resources, such as in mobile devices. In this work, we introduce an algorithm that removes units and layers of a neural network while not changing the output that is produced, which thus implies a lossless compression. This algorithm, which we denote as LEO (Lossless Expressiveness Optimization), relies on Mixed-Integer Linear Programming (MILP) to identify Rectifier Linear Units (ReLUs) with linear behavior over the input domain. By using  $\ell_1$  regularization to induce such behavior, we can benefit from training over a larger architecture than we would later use in the environment where the trained neural network is deployed.

Table 1. Compression of 2-hidden-layer rectifier networks trained on MNIST. Each line summarizes tests on 31 networks. Depending on how the network is trained, the higher incidence of stable units allows for more compression while preserving the trained network accuracy. For example, training with  $\ell_1$  regularization induces such stability and then inactive units can be removed. Interestingly, the small amount of regularization that improves accuracy during training also helps compressing the network later.

Layer	$\ell_1$		
width	weight	Accuracy (%)	Compression (%)
25	0.001	$95.76\pm0.05$	$22 \pm 1$
25	0.0002	$97.24 \pm 0.02$	$8.3\pm0.7$
25	0	$96.68\pm0.03$	$0\pm 0$
50	0.001	$96.05\pm0.04$	$29.4\pm0.7$
50	0.0002	$97.81 \pm 0.02$	$15.1\pm0.6$
50	0	$97.62\pm0.02$	$0\pm 0$
100	0.0005	$97.14 \pm 0.02$	$30.8\pm0.5$
100	0.0001	$98.14\pm0.01$	$14.9\pm0.4$
100	0	$98.00\pm0.01$	$0\pm 0$

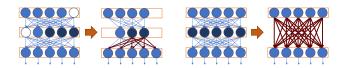


Figure 1. Examples of output-preserving neural network compression obtained with LEO. On the left, two units in white are stably inactive and three units indexed by set S in darker blue are stably active, where rank( $W_S^2$ )=2. In such a case, we can remove the stably inactive units and merge the stably active units to produce the same input to the next layer using only two units. On the right, an entire layer is stably active. In such a case, we can fold the layer by directly connecting the layers before and after it. In both cases, the red arcs correspond to adjusted network coefficients.

<sup>&</sup>lt;sup>1</sup>Bucknell University, Lewisburg, Pennsylvania, USA <sup>2</sup>Michigan State University, East Lansing, Michigan, USA <sup>3</sup>The University of Utah, Salt Lake City, USA. Correspondence to: Thiago Serra <thiago.serra@bucknell.edu>.

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