Relative Fuzzy Connectedness on Directed Graphs and its Application in a Hybrid Method for Interactive Image Segmentation
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To cite this version:
Hans Harley Ccacyahuillca Bejar. Relative Fuzzy Connectedness on Directed Graphs and its Application in a Hybrid Method for Interactive Image Segmentation. LatinX in AI Research at ICML 2019, Jun 2019, Long Beach, California, United States. hal-02244966

HAL Id: hal-02244966
https://hal.archives-ouvertes.fr/hal-02244966
Submitted on 1 Aug 2019

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Abstract

Image segmentation consists of dividing an image into its composing regions or objects, for example, to isolate the pixels of a target object of a given application. In segmentation of medical images, the object of interest commonly presents transitions at its border predominantly from bright to dark or dark to bright. Traditional region-based methods of image segmentation, such as Relative Fuzzy Connectedness (RFC), do not distinguish well between similar boundaries with opposite orientations. The specification of the boundary polarity can help to alleviate this problem but this requires a mathematical formulation on directed graphs. A discussion on how to incorporate this property in the RFC framework is presented in this work. A theoretical proof of the optimality of the new algorithm, called Oriented Relative Fuzzy Connectedness (ORFC), in terms of an energy function on directed graphs subject to seed constraints is presented, and its application in powerful hybrid segmentation methods. The hybrid method proposed ORFC & Graph Cut preserves the robustness of RFC respect to the seed choice, avoiding the shrinking problem of Graph Cut (GC), and keeps the strong control of the GC in the contour delineation of irregular image boundaries. The proposed methods are evaluated using medical images of MRI and CT images of the human brain and thoracic studies.

Theoretical definition of ORFC

ORFC as a directed cut in the digraph

ORFC is supported by a graph cut optimality criterion, which encompasses RFC as a particular case. In the case of directed graphs, we have two possible sets of cuts (inner and outer). Let’s consider the inner cut:

\[ C_0(x) = \{(a, b) \in E : \chi(a) = 0 \land \chi(b) = 1\} \]

So we have the following formulation for the energy functional of the \( \varepsilon_{\infty} \)-minimizing problem:

\[ \varepsilon_{\infty}^w(x) = \max_{(a, b) \in C_0} w(a, b) \]

Let \( \varepsilon_{\min}^w(x) \) be the minimum value of the energy \( \varepsilon_{\infty}^w(x) \), that is:

\[ \varepsilon_{\min}^w(x) = \min_{(a, b) \in C_0} \varepsilon_{\infty}^w(x) \]

We have the following set of solutions:

\[ \chi_{\infty}^w(S_b, S_o) = \{x \in X(S_b, S_o) : \varepsilon_{\infty}^w(x) = \varepsilon_{\min}^w\} \]

The ORFC algorithms on digraphs have the following definitions based on cut in graph

For the inner cut “in” with one internal seed \( s \),

\[ \chi_{\infty}^{A_{ORFC}}(s_1, S_b) = \chi_{\infty}^{A_{ORFC}}(S_b, S_o) = |O| = \min \{|P| : \chi_P \subseteq X_{\infty}^w(s_1, S_b)\} \]

Hybrid segmentation: ORFC & GC

Algorithm to compute \( A_{ORFC+GC}^{\infty}(S_b, S_o) \):

1. Compute \( P \): \( \chi_P = A_{ORFC}^{\infty}(S_o, S_b) \).
2. Compute \( Q \): \( \chi_Q = A_{RFC}^{\infty}(S_b, S_o) \).
3. Compute and return \( A_{ORFC+GC}^{\infty}(P, Q) \).

Experimental Results

ORFC algorithm

Algorithm to compute \( A_{ORFC}^{\infty}(s_1, S_b) \):

1. Compute the value of the map \( V_b(s_1) \) for the function \( f_{\infty}^S \).
2. Remove from the graph \( G \) all edges with weight \( \leq \varepsilon_{\infty}^w \).
3. Assign to the object the pixels that belong to the directed connected component of basepoint \( s_1 \) in the transpose graph of \( G \) (i.e., \( A_{ORFC}^{\infty}(s_1, S_b) = \chi_D \cdot O = DCC_{G^T}(s_1) \)).

Algorithm to compute \( A_{ORFC}^{\infty}(s_1, S_b) \):

1. Compute the value of the map \( V_b(s_1) \) for the function \( f_{\infty}^S \).
2. Remove from the graph \( G \) all edges with weight \( \leq \varepsilon_{\infty}^{\min} \).
3. Assign to the object the pixels that belong to the directed connected component of basepoint \( s_1 \) in the graph \( G \) (i.e., \( A_{ORFC}^{\infty}(s_1, S_b) = \chi_D \cdot O = DCC_G(s_1) \)).

References

Example of \( A_{ORFC+GC}^{\infty}(S_b, S_o) \):

A conference paper was published in SIBGRAPI [HHCB14], and one journal paper was published in the EURASIP Journal on Image and Video Processing [HHCB15].

Calcaneeus

Brain


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