Relative Fuzzy Connectedness on Directed Graphs and its Application in a Hybrid Method for Interactive Image Segmentation

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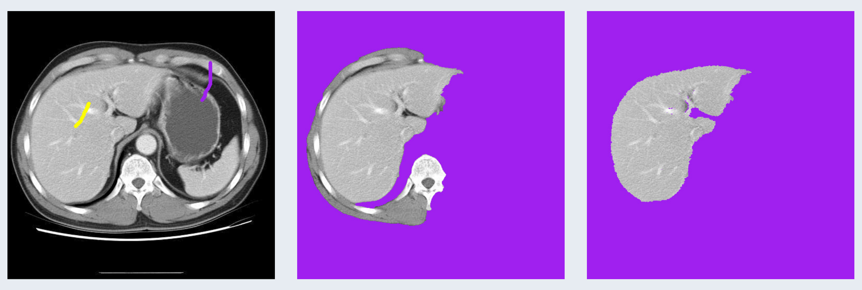
Abstract



Original OIFT ORFC

Image segmentation consists of dividing an image into its composing regions or objects, for example, to isolate the pixels of a target object of a given application. In segmentation of medical images, the object of interest commonly presents transitions at its border predominantly from bright to dark or dark to bright. Traditional region-based methods of image segmentation, such as Relative Fuzzy Connectedness (**RFC**), do not distinguish well between similar boundaries with opposite orientations. The specification of the **boundary polarity** can help to alleviate this problem but this requires a mathematical formulation on **directed graphs**. A discussion on how to incorporate this property in the RFC framework is presented in this work. A theoretical proof of the optimality of the new algorithm, called Oriented Relative Fuzzy Connectedness (**ORFC**), in terms of an **energy function** on directed graphs subject to **seed constraints** is presented, and its application in powerful **hybrid** segmentation methods. The hybrid method proposed **ORFC & Graph Cut** preserves the **robustness** of ORFC respect to the seed choice, avoiding the shrinking problem of Graph Cut (GC), and keeps the strong control of the GC in the **contour delination** of irregular image boundaries. The proposed methods are evaluated using **medical images** of MRI and CT images of the human brain and thoracic studies.

Results

In this work, we introduced the *ORFC* technique and showed that it can effectively exploit the boundary polarity improving the results in relation to its predecessor *RFC*. We also presented a powerful **hybrid approach**, which outperforms the previous works [KCC13, KCC12C].
A conference paper was published in SIBGRAPI [HHCB14], and one **journal paper** was published in the EURASIP Journal on Image and Video Processing [HHCB15].

Theorical definition of ORFC

ORFC as a directed cut in the digraph

ORFC is supported by a **graph cut optimality criterion**, which **encompasses RFC** as a particular case. In the case of directed graphs, we have two possible sets of **cuts (inner and outer)**. Let's consider the inner cut:

$$\mathcal{C}_{in}(x) = \{\langle a, b\rangle \in E : x(a) = 0 \wedge x(b) = 1\}$$

So we have the following formulation for the **energy functional** of the ε_∞-minimizing problem.

$$\varepsilon^{in}_\infty(x) = \max_{\langle a,b\rangle \in \mathcal{C}_{in}} w(a, b)$$

Let $\varepsilon^{in}_{\infty\downarrow}$ be the **minimum value** of the energy $\varepsilon^{in}_\infty(x)$, that is:

$$\varepsilon^{in}_{\infty\downarrow} = \min\{\varepsilon^{in}_\infty(x) : x \in \mathcal{X}(\mathcal{S}_o, \mathcal{S}_b)\}$$

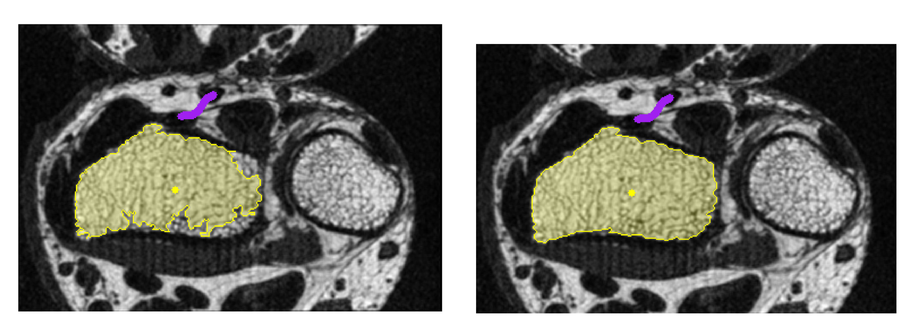
Therefore, we have the following **set of solutions**:

$$\mathcal{X}^{in}_\infty(\mathcal{S}_o, \mathcal{S}_b) = \{x \in \mathcal{X}(\mathcal{S}_o, \mathcal{S}_b) : \varepsilon^{in}_\infty(x) = \varepsilon^{in}_{\infty\downarrow}\}$$

The *ORFC* algorithms on digraphs have the following definitions based on cut in graph:
For the **inner cut "*in*"** with one internal seed s_1,

$$A^{in,✂}_{ORFC}(\{s_1\}, \mathcal{S}_b) = \chi_O \in \mathcal{X}^{in}_\infty(\{s_1\}, \mathcal{S}_b) : |O| = \min\{|P| : \chi_P \in \mathcal{X}^{in}_\infty(\{s_1\}, \mathcal{S}_b)\}$$

Hybrid segmentation: ORFC & GC

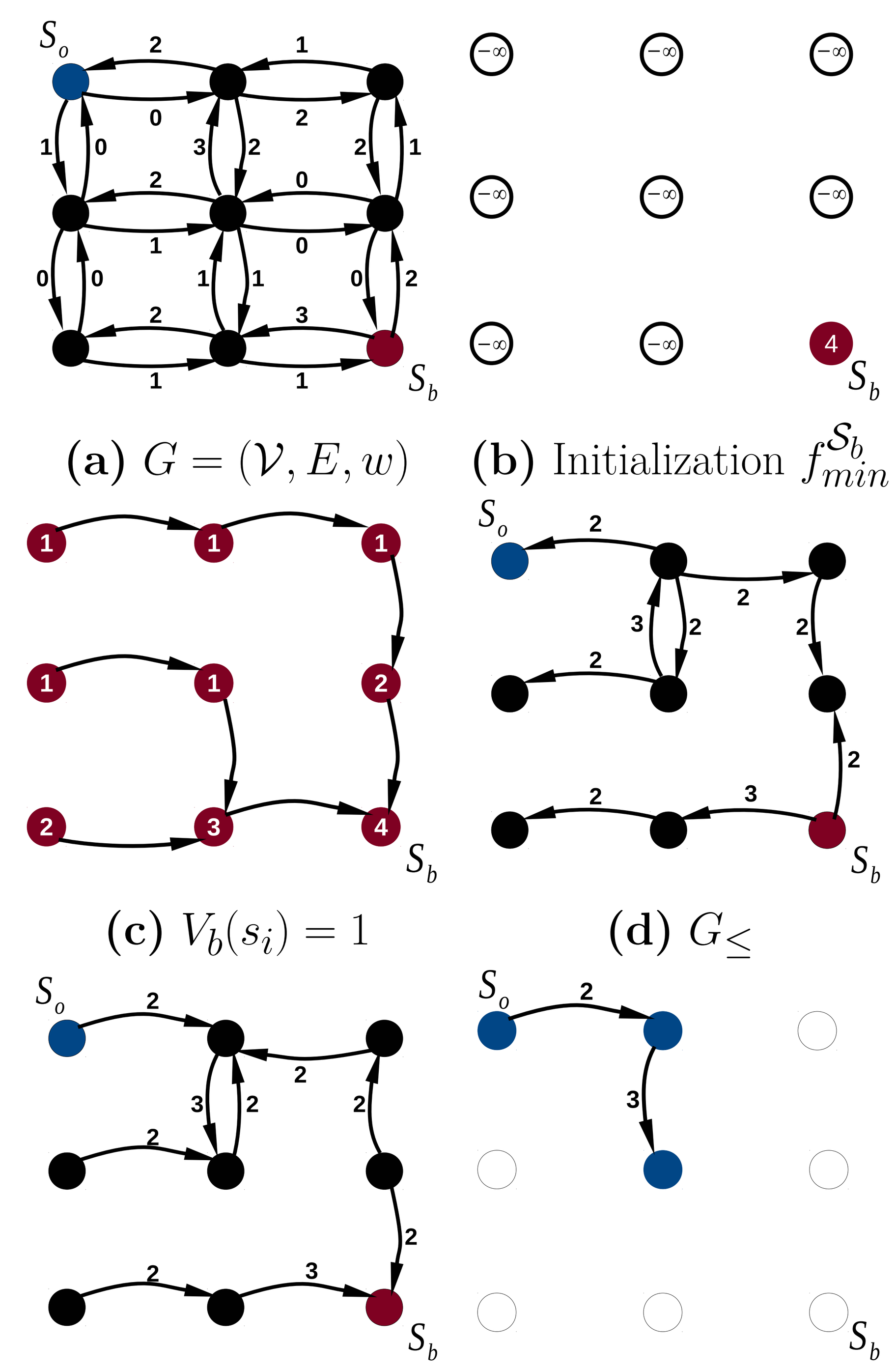


IRFC ORFC+GC

Algorithm to compute $A^{out,✂}_{ORFC+GC}(\mathcal{S}_o, \mathcal{S}_b)$:

1. Compute P: $\chi_P = A^{out,✂}_{ORFC}(\mathcal{S}_o, \mathcal{S}_b)$.
2. Compute Q: $\chi_Q = A^{in,✂}_{ORFC}(\mathcal{S}_b, \mathcal{S}_o)$.
3. Compute and return $A^{out}_{OGC}(P, Q)$.

Example of $A^{in,✂}_{ORFC}(\mathcal{S}_o = \{s_i\}, \mathcal{S}_b)$

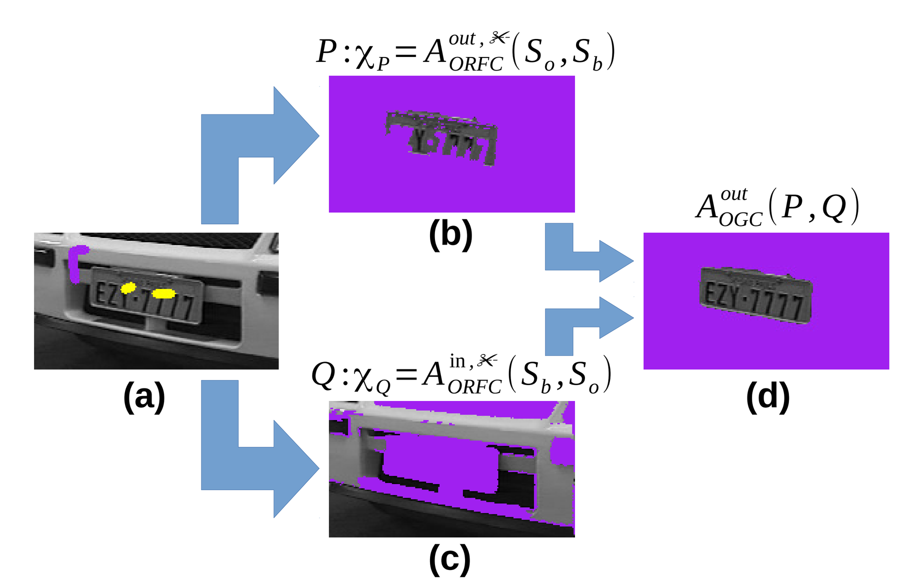


(a) $G = (\mathcal{V}, E, w)$ (b) Initialization $f^{\mathcal{S}_b}_{min}$

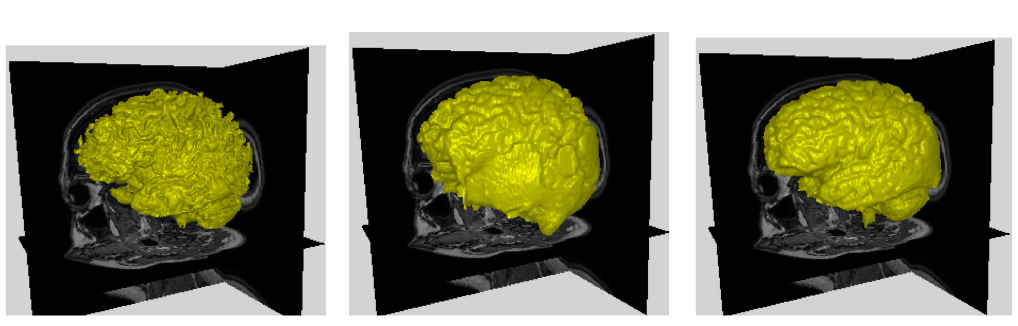
(c) $V_b(s_i) = 1$ (d) $G_\leq$

(e) $G^T_\leq$ (f) Result

Example of $A^{out,✂}_{ORFC+GC}(\mathcal{S}_o, \mathcal{S}_b)$



ORFC algorithm



RFC RFC+GC ORFC+OGC

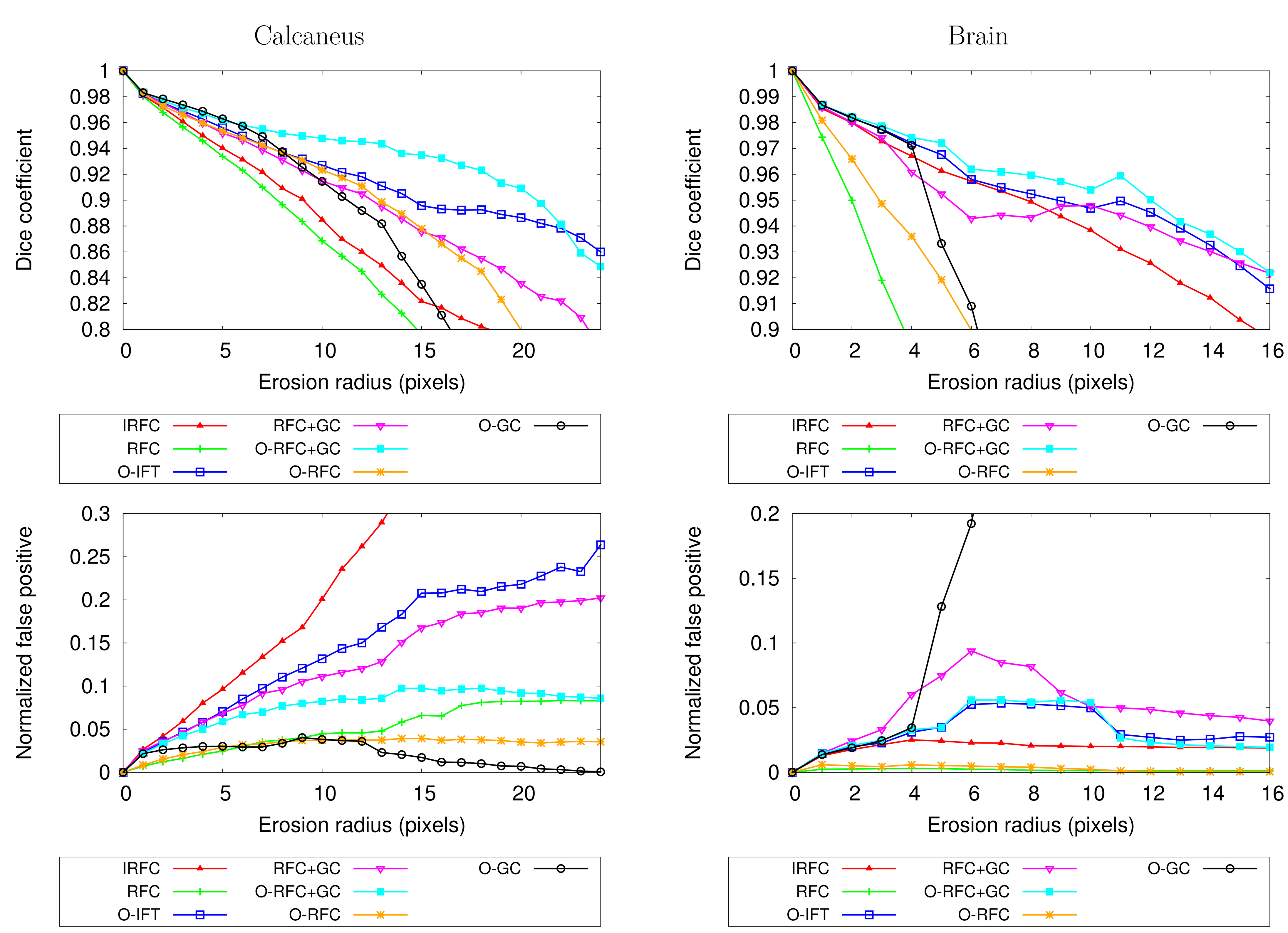
Algorithm to compute $A^{in,✂}_{ORFC}(\{s_i\}, \mathcal{S}_b)$:

1. Compute the value of the map $V_b(s_i)$ for the function $f^{\mathcal{S}_b}_{min}$.
2. Remove from the graph G all edges with weight $\leq \varepsilon^{in}_{\infty\downarrow} = V_b(s_i)$, obtaining a new graph $G_\leq$.
3. Assign to the object the pixels that belong to the directed connected component of basepoint s_i in the transpose graph of $G_\leq$ (i.e., $A^{in,✂}_{ORFC}(\{s_i\}, \mathcal{S}_b) = \chi_O : O = DCC_{G^T_\leq}(s_i)$).

Algorithm to compute $A^{out,✂}_{ORFC}(\{s_i\}, \mathcal{S}_b)$:

1. Compute the value of the map $V^{\not\parallel}_b(s_i)$ for the function $f^{\not\parallel\mathcal{S}_b}_{min}$.
2. Remove from the graph G all edges with weight $\leq \varepsilon^{out}_{\infty\downarrow} = V^{\not\parallel}_b(s_i)$, obtaining a new graph $G_\leq$.
3. Assign to the object the pixels that belong to the directed connected component of basepoint s_i in the graph $G_\leq$ (i.e., $A^{out,✂}_{ORFC}(\{s_i\}, \mathcal{S}_b) = \chi_O : O = DCC_{G_\leq}(s_i)$).

Experimental Results



References

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[HHCB14] Hans H. C. Bejar and Paulo A.V. Miranda. Oriented Relative Fuzzy Connectedness: Theory, Algorithms, and Applications in Image Segmentation. 27th SIBGRAPI Conference on Graphics, Patterns and Images, 304–311, 2014.

[KCC12C] , K.C. Ciesielski and P.A.V. Miranda and J.K. Udupa and A.X. Falcão. Image segmentation by combining the strengths of relative fuzzy connectedness and graph cut. Proc. of the International Conference on Image Processing,